Solution

DPP NO. - 8

1. $\vec{A} = 2\hat{i} + 9\hat{j} + 4\hat{k}$

$4\vec{A} = 8\hat{i} + 36\hat{j} + 16\hat{k}$

2. $\longrightarrow$ (1 m) magnitude & direction must be same.

3. $\frac{dy}{dx} = x.e^x + e^x = (x + 1)\, e^x = 0$; $x = -1$;

$\frac{d^2y}{dx^2} > 0$ for $x = -1$

4. $\frac{dy}{dx} = \frac{d}{dx}(x^5 - 5x^4 + 5x^3 - 10) = 5x^4 - 20x^3 + 15x^2$

$= 0$; $x = 3, 0, 1$

$\frac{d^2y}{dx^2} < 0$ at $x = 1$

5. $\vec{A} = 2\hat{i} + 3\hat{j}$

$\hat{A} = \frac{2\hat{i} + 3\hat{j}}{\sqrt{4+9}} = \frac{2\hat{i} + 3\hat{j}}{\sqrt{13}}$

6*. (B) $\longrightarrow$ (D) $\vec{A}$

7. $A_x = 2$

$A_y = 2\sqrt{3}$

$A = \sqrt{A_x^2 + A_y^2}$

$= \sqrt{4+12} = 4$

8. $\vec{A} = 2\hat{i}$

$\longrightarrow$ 2 units

9. $\vec{B} = 3\,j$ $\uparrow$ 3 units

10. $-4\vec{A} = -8i$ $\longleftarrow$ 8 units

DPP NO. - 9

1. $(\vec{A} + \vec{B}) = 7\hat{i} - 9\hat{j}$

$\therefore \left|\vec{A} + \vec{B}\right| = \sqrt{49 + 81} = \sqrt{130}$

2. unit vector $= \frac{3\hat{i} + 3\hat{j}}{\sqrt{3^2 + 3^2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

3. Apply triangle law of vector addition.

5. $(A^2 + B^2 + 2AB\cos\theta) = \frac{1}{4}(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow 3A^2 + 3B^2 + 10\,AB\cos\theta = 0$

or $12B^2 + 3B^2 + 10(2B)(B)\cos\theta = 0$

$15B^2 + 20B^2\cos\theta = 0$

$\cos\theta = -\frac{3}{4}$

6. Since $\vec{B} = 3\vec{A}$, so both are parallel.

7. Velocity = (speed) $\hat{A}$

$= 6\frac{(2\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{4+4+1}} = (4\hat{i} + 4\hat{j} - 2\hat{k})$ units.

8. $\vec{P} - \vec{Q} = (\hat{i} + \hat{j} - \hat{k}) - (\hat{i} - \hat{j} + \hat{k}) = 2\hat{j} - 2\hat{k}$

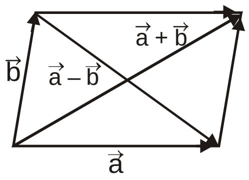
$\therefore$ unit vector along

$\vec{P} - \vec{Q} = \frac{(\vec{P} - \vec{Q})}{|\vec{P} - \vec{Q}|} = \frac{2\hat{j} - 2\hat{k}}{\sqrt{(2)^2 + (-2)^2}}$

$$\therefore \quad \vec{P}-\vec{Q} = \frac{(\vec{P}-\vec{Q})}{|\vec{P}-\vec{Q}|} = \frac{2\hat{j}-2\hat{k}}{\sqrt{(2)^2+(-2)^2}}$$

$$= \frac{2\hat{j}-2\hat{k}}{\sqrt{4+4}} = \frac{2\hat{j}-2\hat{k}}{2\sqrt{2}} = \frac{\hat{j}-\hat{k}}{\sqrt{2}}$$

9.

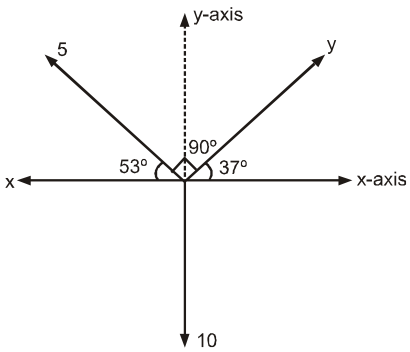


$$\left|\vec{a}+\vec{b}\right| \geq \left|\vec{a}-\vec{b}\right|$$

$\Rightarrow$ angle between $\vec{a}$ & $\vec{b}$ $\leq 90°$

$\Rightarrow$ $\vec{a}$ $\vec{b}$ $\leq 90°$

10.



$\Sigma\vec{F} = 0$

$\Rightarrow$ (y cos37° $\hat{i}$ + y sin37° $\hat{j}$) + (5 cos53°($-\hat{i}$) + 5 sin53° $\hat{j}$) + (x($-\hat{i}$) + 10($-\hat{j}$)) = 0

$$\Rightarrow \left(\frac{4y}{5}-3-x\right)\hat{i}+\left(\frac{3y}{5}+4-10\right)\hat{j} = 0\hat{i}+0\hat{j}$$

Comparing coefficients of $\hat{i}$ & $\hat{j}$ both sides–

$\frac{4y}{5} - x = 3$ ......(i)

$\frac{3y}{5} = 6 \quad \Rightarrow \quad y = 10$

Putting 8 – x = 3 $\quad \Rightarrow \quad$ x = 5

DPP NO. - 10

1. $S_t + S_{t+1} = 100$

$u + \frac{1}{2}f(2t-1) + u + \frac{1}{2}f[2(t+1)-1] = 100$

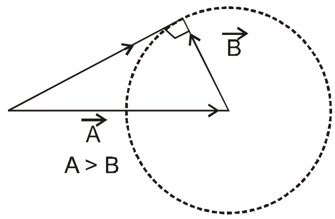
$2u + \frac{1}{2}f(2t-1+2t+1) = 100$

2u + 2ft = 100

u + ft = 50

v = 50 cm/s.

2.



So, A > B

3. time taken by car to cover first half distance.

$= \frac{1}{40}$ hr $= \frac{1}{40} \times 60$ min = 1.5 min.

Remaining time = 2.5 – 1.5 = 1 min.

required speed $= \frac{1\text{km}}{1\text{min}}$ = 60 km/hr

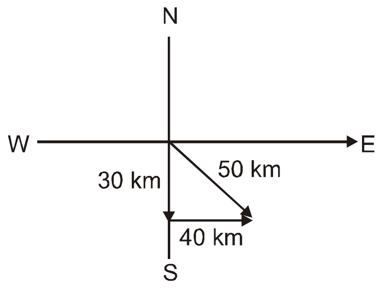
4. $r = \sqrt{a^2-t^2} + t \cos t^2$

$V = \frac{dr}{dt} = \frac{1}{2}(a^2-t^2)^{-1/2}(-2t) + t(-\sin t^2)\,2t.$

$+ \cos t^2.$

$V = -\frac{t}{\sqrt{a^2-t^2}} - 2t^2 \sin t^2 + \cos t^2.$

5.



Net displacement = 50 km

6. $\sqrt{x} = (2t - 3)$ for B option

$x = (2t - 3)^2$ accelerated for $t > 3/2$

$$\frac{dx}{dt} = 2(2t - 3)\,(2) = 4(2t - 3)$$

$V = 4(2t - 3) = 0$

rest at $t = 3/2$

$a = 8$ m/s.

7. since $\frac{\text{Distance}}{\Delta t} \geq \frac{|\text{Displacement}|}{\Delta t}$

aV speed $\geq$ | aV. velocity |

in uniform circular motion speed is constant but acc. $\neq 0$

in uniform circle motion after one round average velocity becomes zero.

8. Let u be initial velocity & a be its acceleration

Distance in first 2 sec = S_1 = 200 cm

$\Rightarrow\ u(2) + \frac{1}{2}a(2)^2 = 200$ cm

$\Rightarrow\ u + a = 100$ ....(i)

Distance in next 4 sec. = S_2 = 220 cm

Distance in first 6 sec. = $S_1 + S_2$ = 200 + 220 cm

$\Rightarrow\ u(6) + \frac{1}{2}a(6)^2 = 420$

$\Rightarrow\ u + 3a = 70$ ....(ii)

From equations (i) & (ii), we get

$a = -15$ cm/s^2 , $u = 115$ cm/s

Hence, velocity at the end of 7 sec. from start

= $u + 7a$

= $115 + 7(-15)$ = 10 cm/s.

DPP NO. - 11

1. Let u be velocity of ball with which it is thrown.

$h = ut + (-\frac{1}{2}gt^2)$ $\quad 25 = ut - 5t^2$

$5t^2 - ut + 25 = 0$ Let t_1 , t_2 be its roots

$t_1 + t_2 = u/5,$ $\quad t_1t_2 = 5$

Given, $t_2 - t_1 = 4$ sec.

$(t_2 - t_1)^2 = 16$

$\Rightarrow\ (t_2 + t_1)^2 - 4t_1t_2 = 16$

$\left(\frac{u}{5}\right)^2 - 4 \times 5 = 16$ $\quad u = 30$ m/sec.

2. For a freely falling body

$S = \frac{1}{2}gt^2$ $\quad S \propto t^2$.

3. v(2) = v(0) + area under a–t graph from t = 0 to t = 2

$= 2 + \frac{1}{2}(2)\,(4) = 6$ m/s.

4. Distance covered in first 10 sec

$S_i = \frac{1}{2}(10)\,(10)^2 = 500$ m

Remaining height from ground = 2495 – 500 = 1995 m

$u = gt = 10 \times 10 = 100$ m/s velocity on reaching the ground

$v^2 = (100)^2 + 2(-2.5) \times 1995$

$v^2 = 10000 - 9975 = 25$

$v = 5$ m/s.

5. Suppose the particle starts from origin at t = 0. Then at any time t,

$x \propto t^3$

$x = kt^3$ (K = constant)

$$v = \frac{dx}{dt} = 3kt^2$$

$$a = \frac{dv}{dt} = 6kt$$

$a \propto t$.

6. Displacement = 0 ($\because$ initial position = final position)

average velocity = 0 ($\because$ Total displacement = 0)

7. $V = (3t^2 - 18t + 24)$ m/s

$V = 3(t-2)(t-4)$

$$s = \left|\int_0^2 V dt\right| + \left|\int_2^3 V dt\right|$$

$$= \left|\int_0^2 (3t^2 - 18t + 24)dt\right| + \left|\int_2^3 (3t^2 - 18t + 24)dt\right| =$$

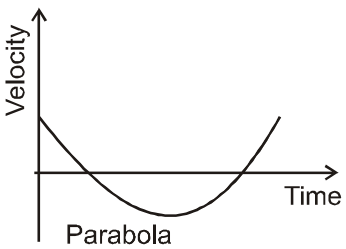
$|20| + |-2| = 22$ m

8. $V = 3(t-2)(t-4)$

$a = 6(t-3)$

common interval in which V and a both have opposite sign is 0 to 2 sec

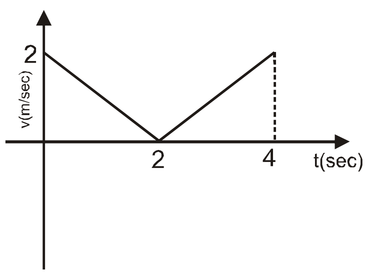
9. Velocity time graph will be



Speed time graph = |Velocity time graph|

DPP NO. - 12

1. Plotting velocity v against time t, we get

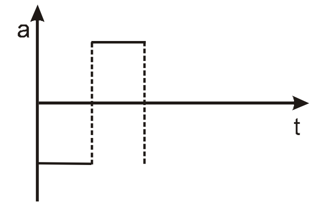


Area under the v–t curve gives distance.

Distance = $\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2 = 4$m

2. Obviously slope of v–t graph is changed at t = 2, 4,6,......... in direction but it has constant magnitude.

3. Instantaneous, acceleration = slope of v–t graph hence, obviously, a – t graph will be as shown,



4. **(A)**

$\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt} = (2t-4)\hat{i} + 2t\hat{j}$

, $\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$

if $\vec{a}$ and $\vec{v}$ are perpendicular

$\vec{a} \cdot \vec{v} = 0$

$(2\hat{i} + 2\hat{j}).((2t-4)\hat{i} + 2t\hat{j}) = 0$

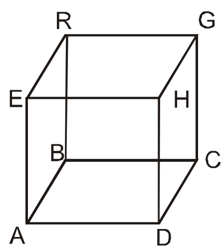
$8t - 8 = 0$

t = 1 sec.

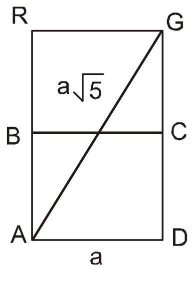
Ans. t = 1 sec.

5. $$\frac{S_N}{S} = \frac{\frac{1}{2}a(2n-1)}{\frac{1}{2}a\,n^2} = \frac{2n}{n^2} - \frac{1}{n^2} \qquad \frac{2}{n} - \frac{1}{n^2}$$

6.



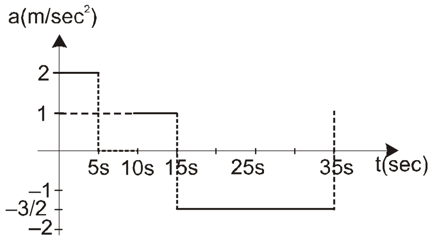
on placing back face and bottom face in same plane.

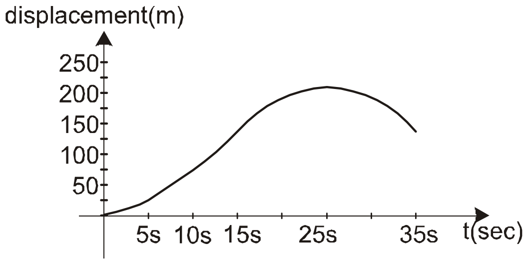


A → starting point G → final point

minimum time = $\frac{\sqrt{5}a}{u}$

7.





Maximum displacement is a 25 sec. displacement = 25 + 50 + 62.5 + 75 = 212.5 m.

8. (i) **Impossible:** Speed is always positive
(ii) **Impossible:** Time never decreases.
(iii) **Possible:** Velocity may increase with time.

DPP NO. - 13

1. At t = 4 sec, V = 0 + (4) (4) = 16 m/sec.

At t = 8 sec, V = 16 m/sec.

At t = 12 sec, V = 16 – 4 (12 – 8) = 0

For 0 to 4 sec ; $s_1 = ½ at^2 = ½ (4) (4)^2 = 32$ m

For 4 to 8 sec ; $s_2 = 16 (8 – 4) = 64$ m

For 8 to 12 sec ; $s_3 = 16 (4) – ½ (4) (4)^2 = 32$ m

So $s_1 + s_2 + s_3 = 32 + 64 + 32 = 128$ m

Alter : Draw v-t graph

Area of v-t graph = displacement.

2. Using $v_x = u_x + a_x t$
= 4 i + (2i) 4
= 12 i
As $a_y = 0$, velocity component in y-direction remains unchanged. Final velocity = 12 i - 5j

speed at t = 4 sec. = $\sqrt{12^2 + (-5)^2}$ = 13 m/s.

$v_x = u_x + a_x t$
= 4 i + (2i) 4
= 12 i

4. V = a + bx
(V increases as x increases)

$\frac{dV}{dx} = b; \quad \frac{dx}{dt} = V$

so, acceleration = $V \frac{dV}{dx}$ = V.b

hence acceleration increases as V increases with x.

5. The retardation is given by

$$\frac{dv}{dt} = -av^2$$

integrating between proper limits

$$\Rightarrow -\int_u^v \frac{dv}{v^2} = \int_0^t a\,dt$$

$$\text{or} \quad \frac{1}{v} = at + \frac{1}{u}$$

$$\Rightarrow \frac{dt}{dx} = at + \frac{1}{u}$$

$$\Rightarrow dx = \frac{u\ dt}{1+aut}$$

integrating between proper limits

$$\Rightarrow \int_0^s dx = \int_0^t \frac{u\,dt}{1+aut}$$

$$\Rightarrow S = \frac{1}{a} \ln(1 + aut)$$

Sol. 6 to 8

The velocity of particle changes sign at t = 1 sec.

∴ Distance from t = 0 to t = 2 sec. is

$$= \int_1^0 v\,dt + \int_2^1 v\,dt$$

$$= \left[(t^3 - \frac{3}{2}t^2)\right]_1^0 + \left[(t^3 - \frac{3}{2}t^2)\right]_2^1 = 3 \text{ m}$$

Displacement from t = 0 to t = 2 sec. is $\int_0^2 v\,dt$

$$= \left[(t^3 - \frac{3}{2}t^2)\right]_0^2 = 2 \text{ m.}$$

DPP NO. - 14

1. m = 2kg, $\vec{F} = \hat{i} - \hat{j}$.

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{1}{2}(\hat{i} - \hat{j})$$

Now $\vec{V} = \vec{u} + \vec{a}$ t.

$$\Rightarrow \vec{V} = 2\hat{i} + \frac{1}{2}(\hat{i} - \hat{j})\,t.$$

$$= \left(2 + \frac{t}{2}\right)\hat{i} - \frac{t}{2}\hat{j} = \frac{1}{2}(t + 4)\,\hat{i} - \frac{t}{2}\hat{j}.$$

Alter : Substitute t = 0 in option and get answer

2. $x^2 = t^2 + 1$

$$2x\frac{dx}{dt} = 2t$$

$\Rightarrow$ xV = t

$xa + V^2 = 1$

$$a = \frac{1 - V^2}{x} = \frac{1 - \frac{t^2}{x^2}}{x}$$

$$\Rightarrow a = \frac{x^2 - t^2}{x^3} = \frac{1}{x^3}$$

3. 54 km/h = 54 × $\frac{5}{18}$ = 15 m/s

$$\langle a \rangle = \frac{15 - (-15)}{10} = 3 \text{ m/s}^2.$$

4. For minimum number of jumps, range must be maximum.

$$\text{maximum range} = \frac{u^2}{g} = \frac{(\sqrt{10})^2}{10} = 1 \text{ meter.}$$

Total distance to be covered = 10 meter

So total step = 10

5. From 6:00 AM to 6:30 AM

displacement of tip of minute hand

= 2 × 10cm = 20 cm

Hence, average velocity = $\frac{20 \text{ cm}}{30 \text{ min}} = \frac{2}{3}$ cm min^{-1}.

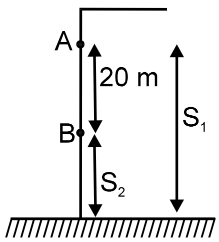
6. Vel. of Ist stone when passing at A $\rightarrow$

$V^2 = 0 + 2.10.5$

V = 10 m/s

$S_1 - S_2 = 20$ m.

$$\Rightarrow \left(10.t + \frac{1}{2}10.t^2\right) - \left(\frac{1}{2}.10.t^2\right) = 20$$



t = 2s

$$S_2 = \frac{1}{2}.10.4 = 20 \text{ m}$$

Ht = 25 + 20 = 45 m.

7. $$\cos\theta = \frac{(\sqrt{3}\,\hat{i} + \sqrt{2}\,\hat{j} - 2\hat{k})(-\hat{j})}{\sqrt{3 + 2 + 4}\,(1)} = \frac{-\sqrt{2}}{3}$$

$$\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right) \quad \text{or} \quad \pi - \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

8. $$\frac{dv}{dt} = g - kv \qquad \int_0^v \frac{dv}{g - kv} = \int_{t=0}^{t} dt$$

$$-\frac{1}{k}\ln\left(\frac{g - kv}{g}\right) = t$$

$g - kv = ge^{-kt}$ $\quad v = \frac{g}{k}\left[1 - e^{-kt}\right]$

$a = \frac{g}{k}\left[0 - e^{-kt}(-k)\right]$

$= g\,e^{-kt}$

$V = \frac{g}{k} - \frac{a}{k} = -\frac{a}{k} + \frac{g}{k}$

$V - \frac{g}{k} = -\frac{a}{k}$

$kv - g = -a$

$a = g - kv$

$= -kv + g$

9. (i) $V\frac{dv}{dx} = -\beta V$ (ii) $a = -\beta V$

$dv = -\beta dx$ $\quad \frac{dv}{dt} = -\beta V$

$\int_{v_0}^{0} dv = -\beta \int_{0}^{x} dx$ $\quad \int_{v_0}^{v} \frac{dv}{v} = -\beta \int_{0}^{t} dt$

$-v_0 = -\beta x$ $\quad \ell n\left(\frac{V}{V_0}\right) = -\beta t$

$x = \frac{v_0}{\beta}$ $\quad V = V_0 e^{-\beta t}$

$V = \frac{V_0}{e^{\beta t}}$ $\quad$ at $t \to \infty$ $V = 0$.

$\therefore$ A & B are correct answer

10. $u = +8$ m/s

$a = -4$ m/s^2

$v = 0$

$\Rightarrow$ $0 = 8 - 4t$ or $t = 2$ sec.

displacement in first 2 sec.

$S_1 = 8 \times 2 + \frac{1}{2}.(-4).\,2^2 = 8$ m

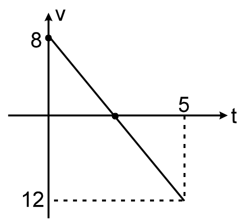
displacement in next 3 sec.

$S_2 = 0 \times 3 + \frac{1}{2}(-4)3^2 = -18$ m.

distance travelled $= |S_1| + |S_2| = 26$ m.

Ans. 26 m.

ALITER :



total distance $= \frac{1}{2} \times 2 \times 8 + \frac{1}{2} \times 3 \times 12$

$= 8 + 18 = 26$ m

DPP NO. - 15

2. At maximum height $v = u\cos\theta$

$\frac{u}{2} = v \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(120°)}{g}$

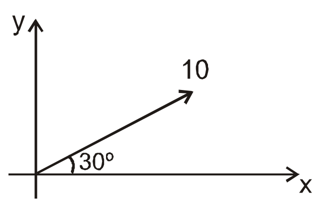
$= \frac{u^2 \cos 30°}{g} = \frac{\sqrt{3}\,u^2}{2g}$

3. At the top of trajectory,

$K' = \frac{1}{2}m(u\cos\theta)^2$

$= \frac{1}{2}mu^2.\cos^2 45° = \frac{k}{2}$.

4. For A



Velocity of the particle will be perpendicular to the initial direction when $10 - g\sin 30°\, t = 0$

$\therefore t = 2$ s,

but total time of flight $= \frac{2u\sin 30^\circ}{g} = 1$ s.

So not possible

For B

Minimum speed during the motion is

$= u \cos 30^\circ = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ m/s.

For B

$t = \frac{1}{2}$ second

$\therefore$ particle is at highest point.

where, displacement $= \sqrt{\frac{R^2}{4} + H^2} = \frac{5\sqrt{13}}{4}$ m

5. For maximum range, $\theta = 45^\circ$

At the highest point, $v = u \cos\theta = \frac{u}{\sqrt{2}}$

6. Range is same for 2θ and 4θ.

$\therefore \quad 2\theta + 4\theta = 90^\circ \quad \Rightarrow \quad \theta = 15^\circ$

$\therefore$ Ratio of ranges will be sin30º : sin 60º : sin120º.

$\frac{1}{2} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}}{2} \Rightarrow \frac{2}{\sqrt{3}} : 2 : 2$

7. $y = u_x t - \frac{1}{2}.g t^2 \quad = 10 \times 1 - 5 \times 1^2 = 5$ m

$x = u_x t \quad = 10 \times 1 = 10$ m

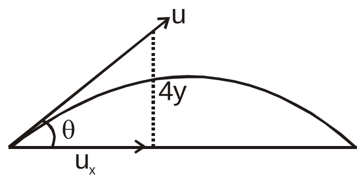
8. For constant acceleration if initial velocity makes an oblique angle with acceleration then path will be parabolic.

DPP NO. - 16

1. $y = x \tan\theta \left(1 - \frac{x}{R}\right)$ $y = (12\,x)\left(1 - \frac{x}{16}\right)$

$\Rightarrow$ Range = **16 m Ans.**

2.



$y = 4t - t^2$, $x = 3t$

$V_y = \frac{dy}{dt} = 4 - 2t$, $V_x = \frac{dx}{dt} = 3$

$\Rightarrow u_y = v_y \big|_{t=0} = 4,\ u_x = v_x \big|_{t=0} = 3$

The angle of projection :

$\tan\theta = \frac{V_y}{V_x} = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$ **Ans.**

3. $V_A \sin 60^\circ = V_B$

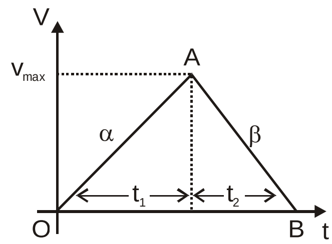
$\Rightarrow \frac{V_A}{V_B} = \frac{2}{\sqrt{3}}$

4. $t = t_1 + t_2$

slope of OA curve $= \tan\theta = \alpha = \frac{v_{max}}{t_1}$

slope of AB curve $= \beta = \frac{v_{max}}{t_2}$

$t = t_1 + t_2$



$\Rightarrow t = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} \Rightarrow v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right) t$

5. The velocity of an object released in a moving frame is equal to that of the frame as observed from the frame.

6. velocity of ball w.r.t. ground = 20 – 10 = 10 m/sec upwards.

$x = ut + \frac{1}{2}at^2$

$120 = -10t + \frac{1}{2} \times 10t^2$

$24 = -2t + t^2$

$t^2 - 2t - 24 = 0$

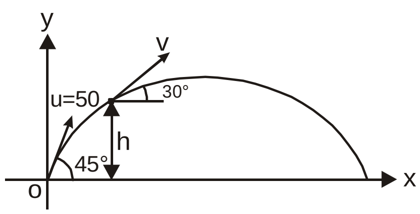
$t = 6$ sec.

7. $\frac{H}{R} = \frac{\tan\theta}{4}$

$\theta = 45°$ & $R = 36$ m

$H = 9$ m

8.



h = height of the point where velocity makes 30º with horizontal.

As the horizontal component of velocity remain same

$50\cos 45° = v\cos 30°$

$v = 50\sqrt{\frac{2}{3}}$

Now by equation

$v^2 = u^2 + 2a_y y$

$\left(50 \times \sqrt{\frac{2}{3}}\right)^2 = 50^2 - 2g \times h$

$\Rightarrow 2gh = 50^2 - 50^2 \times \frac{2}{3}$

$\Rightarrow 2gh = \frac{1}{3} \times 50^2$

$\Rightarrow h = \frac{2500}{60} = \frac{125}{3}$

$h = \frac{125}{3}$ m above point of projection

9. (A) $R = \frac{u^2 \sin 2\theta}{g} = \frac{100\sqrt{3}}{2(10)} = 5\sqrt{3}$m

(B) $11.25 = -10\sin 60^o\, t + \frac{1}{2}(10)t^2$

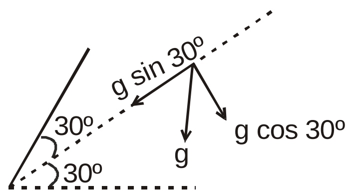
$\Rightarrow 5t^2 - 5\sqrt{3}\,t - 11.25 = 0$

$t = \frac{5\sqrt{3} \pm \sqrt{25(3) + 4(5)(11.25)}}{10}$

$= \frac{5\sqrt{3} \pm \sqrt{3}(10)}{10}$

$= \frac{15}{10}\sqrt{3} = \frac{3}{2}\sqrt{3}$

$R = 10\cos 60\left(\frac{3}{2}\sqrt{3}\right) = 7.5\sqrt{3}$ m



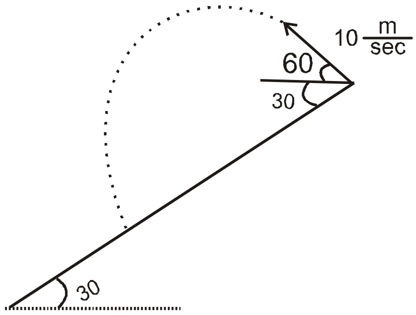
(C) $t = \frac{2u\sin 30^o}{g\cos 30^o} = \frac{2(10)\left(\frac{1}{2}\right)}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$ sec.

$R = 10\cos 30^o\, t - \frac{1}{2} g\sin 30^o\, t^2$

$= \frac{10\sqrt{3}}{2}\left(\frac{2}{\sqrt{3}}\right) - \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{4}{3}$

$= 10 - \frac{10}{3} = \frac{20}{3}$ m

(D) $T = \frac{2(10)}{g\cos 30} = \frac{2(10)}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$ sec.

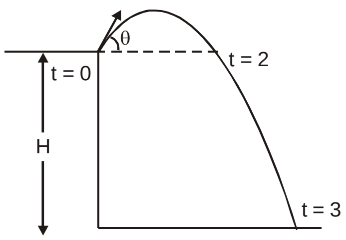


$R = \frac{1}{2}$ g sin 30° t^2

$= \frac{1}{2}(10)\left(\frac{1}{2}\right)\frac{16}{3} = \frac{40}{3}$ m

DPP NO. - 17

1. $2 = \frac{2u_y}{g} \Rightarrow u_y = 10$ m/s



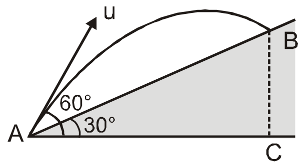
Now, $H = -u_y t + \frac{1}{2}gt^2$

$= -30 + 45 = 15$ m.

3. The horizontal displacement in time t is

AC = u cos 60° t = $\frac{ut}{2}$

$\therefore$ Range on inclined plane = $\frac{AC}{\cos 30} = \frac{ut}{\sqrt{3}}$



4. $V = x^2 + x$

$a = V\frac{dv}{dx} = (x^2 + x)(2x + 1)$

At x = 2 m

a = (4 + 2) (4 + 1)

a = 30 m/s^2 .

6. $x_A = x_B$

$10.5 + 10t = \frac{1}{2}at^2 \quad a = \tan 45° = 1$

$t^2 - 20t - 21 = 0 \qquad t = \frac{20 \pm \sqrt{400 + 84}}{2}$ t = 21 sec.

7. $S_1 - S_2 = 125$ m if $S_1 > S_2$ then

$50\,t - \frac{1}{2} \times 10\,t^2 = 125$

$10\,t - t^2 = 25$

$t^2 - 10\,t + 25 = 0$

t = 5 sec.

$S_2 - S_1 = 125$m if $S_2 > S_1$ then,

$\frac{1}{2} \times 10\,t^2 - 50\,t = 125$

$t^2 - 10\,t - 25 = 0$

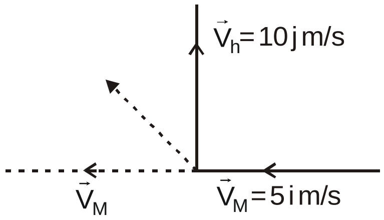
$t = \frac{10 + \sqrt{100 + 100}}{2}$

$t = 5\left(1 + \sqrt{2}\right)$ sec.

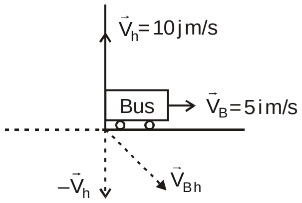
(8 to9) $\vec{V}_{hM} = \vec{V}_h - \vec{V}_M = 10\,j - 10i = -10i + 10\,j$

$\therefore \vec{V}_{hM} = 10(-i) + 10\,j$ $\therefore$ As seen bny

the monkey helicopter is moving in (↖) direction.



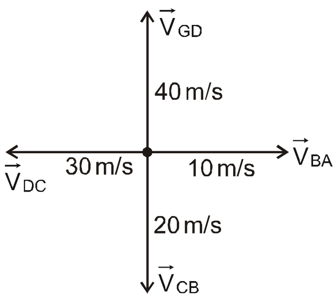
$\vec{V}_{Bh} = \vec{V}_B - \vec{V}_h$ = 15 i – 10 j = 15 i + 10 (j)



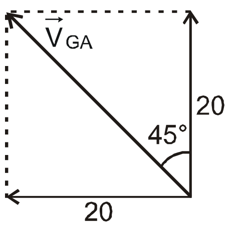
$\therefore$ As seen by helicopter's pilot the bus is moving in (↘) direction.

DPP NO. - 18

1. All the velocities are marked in diagram where G represents ground



adding we get



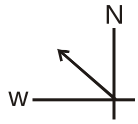
Then $\vec{V}_{GD} + \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = \vec{V}_{GA} = -\vec{V}_{AG}$

Hence velocity of A is towards south east.

2. Vboat , river = $4\hat{i}$

Vriver , ground = $2\hat{i}$

Vwind , ground = $6\hat{j}$

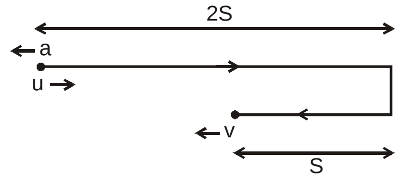


$\vec{V}$ wind, boat $= \vec{V}_{wg} + \vec{V}gr + \vec{V}_{rb} = 6\hat{j} - 2\hat{j} - 4\hat{i}$

$= -4\hat{i} + 4\hat{j}$

so flag blown in north west.

3. Let u and v denote initial and find velocity, then then nature of motion is indicated in diagram



Hence initial and final speed are given by equation

$0^2 = u^2 - 2a \times 2S$ and $v^2 = 0^2 + 2as$

$\therefore \quad v = \frac{u}{\sqrt{2}}$ or $\frac{u}{v} = \sqrt{2}$ **Ans.**

4. $\vec{V}_{O,M} = \vec{V}_O - \vec{V}_M \qquad \vec{V}_{O,M} = \vec{V}_O - \vec{V}_{Train}$

$V_{O,M}$ = velocity of object with respect to man

V_O = velocity of object

V_M = velocity of man

Here velocity of object is zero.

So, $\vec{V}_{O,M} = -\vec{V}_M$

5. If $|\vec{a} \times \vec{u}| = 0$ particle will not follow curved path.

Above described motion is a projectile motion with parabolic path

6. At maximum height, velocity = 0

$H = \frac{u^2}{2g}$ &

At height h = H/2 $\quad V^2 = u^2 - 2gh$

$V^2 = u^2 - 2g.\frac{u^2}{4g} \qquad V^2 = \frac{u^2}{2} \quad \Rightarrow \quad V = \frac{u}{\sqrt{2}}$

Time taken to rise to maximum height T = $\frac{u}{g}$

for height h = $\frac{H}{2}$ t = $\frac{(u - u/\sqrt{2})}{g}$ = $\frac{(\sqrt{2}-1)u}{\sqrt{2}g}$

Time taken to rise to $\frac{3}{4}$ H = T – time taken to fall

down by $\frac{H}{4}$

= T – $\frac{T}{2}$ = $\frac{T}{2}$

7. Let velocity of bodies be v_1 and v_2.
in first case
$u_1 = v_1 + v_2$ .... (i)
in second case
$u_2 = v_1 - v_2$ .... (i)

$\therefore$ $v_1 = \frac{u_1 + u_2}{2}$ and $v_2 = \frac{u_1 - u_2}{2}$

Here $u_1 = \frac{16}{10}$ m/s and $u_2 = \frac{3}{5}$ m/s

After solving we have
v_1 = 1.1 m/s and v_2 = 0.5 m/s.

8. The initial velocity of A relative to B is $\vec{u}_{AB} = \vec{u}_A - \vec{u}_B$

= $(8\hat{i} - 8\hat{j})$ m/s

$\therefore$ $u_{AB} = 8\sqrt{2}$ m/s
Acceleration of A relative to B is -

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = (-2\hat{i} + 2\hat{j})$ m/s^2

$\therefore$ $a_{AB} = 2\sqrt{2}$ m/s^2
since B observes initial velocity and constant acceleration of A in opposite directions, Hence B observes A moving along a straight line.
From frame of B

Hence time when $v_{AB} = 0$ is t = $\frac{u_{AB}}{a_{AB}}$ = 4 sec.

The distance between A & B when $v_{AB} = 0$ is S = $\frac{u_{AB}^2}{2a_{AB}}$ = $16\sqrt{2}$ m

The time when both are at same position is -

T = $\frac{2u_{AB}}{a_{AB}}$ = 8 sec.

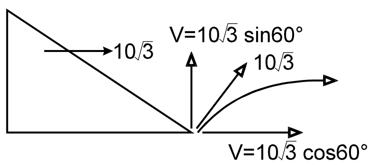
Magnitude of relative velocity when they are at same position in u_{AB} = $8\sqrt{2}$ m/s.

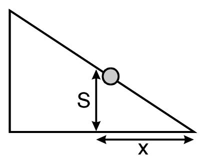
DPP NO. - 19

2. In (A) $x_f - x_i$
0 – x = – x = – ve
So average velocity is – ve.

3. From the graph ; we observe that slope is non–zero positive at t = 0 & slope is continuously decreasing with time and finally becomes zero. Hence we can say that the particle starts with a certain velocity, but the motion is retarded (decreasing velocity)

4. Suppose particle strikes wedge at height 'S' after time t. S = 15t – $\frac{1}{2}$10 t^2 = 15t – 5 t^2. During this time distance travelled by particle in horizontal direction = $5\sqrt{3}$ t. Also wedge has travelled travelled extra distance





x = $\frac{S}{\tan 30^\circ}$ = $\frac{15t - 5t^2}{1/\sqrt{3}}$

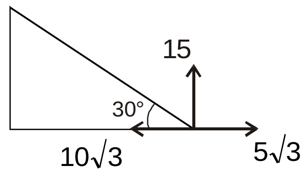
Total distance travelled by wedge in time

t = $10\sqrt{3}$ t. = $5\sqrt{3}$ t + $\sqrt{3}$ (15 – 5t^2)

$\Rightarrow$ t = 2 sec.

Alternate Sol.

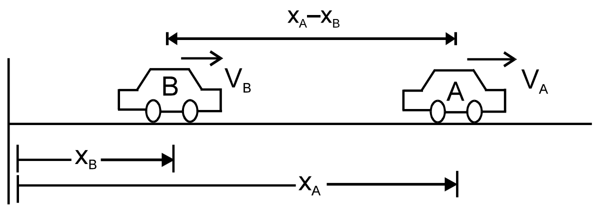
(by Relative Motion)



$$T = \frac{2u\sin 30^\circ}{g\cos 30^\circ} = \frac{2x10\sqrt{3}}{10} \times \frac{1}{\sqrt{3}} = 2 \text{ sec.}$$

$\Rightarrow$ t = 2 sec.

5.



As given

$(V_A - V_B) \propto x_A - x_B$

$(V_A - V_B) = K(x_A - x_B)$

when $x_A - x_B = 10$ We have $V_A - V_B = 10$

We get

10 = K10 $\Rightarrow$ K = 1

$\Rightarrow V_A - V_B = (x_A - x_B)$.........(1)

Now Let

$x_A - x_B = y$ ....(2)

On differentiating with respect to 't' on both side.

$\Rightarrow \frac{dx_A}{dt} - \frac{dx_B}{dt} = \frac{dy}{dt} \Rightarrow V_A - V_B = \frac{dy}{dt}$ .... (3)

$\Rightarrow$ Using (1) , (2), (3)

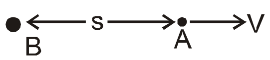
We get $\frac{dy}{dt} = y$

Here y represents sepration between two cars

$$\Rightarrow \int_{10}^{20} \frac{dy}{y} = \int_0^t dt \quad \Rightarrow \quad [\log_e y]_{10}^{20} = t$$

t = ($\log_e 2$) sec ***Required Answer.***

Alter.



(Assume to be at rest)

$V \propto s$

V = ks

V = 10, s = 10, k = 1

$$\frac{ds}{dt} = s \qquad \int_{10}^{20} \frac{ds}{s} = \int_0^t dt$$

6 to 8. At t = 2 sec (t = 2 sec i j)

$v_x = u_x + a_x t = 0 + 10 \times 2 = 20$ m/s

$v_y = u_y + a_y t = 0 - 5 \times 2 = -10$ m/s

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (-10)^2} = 10\sqrt{5} \text{ m/s}$$

From t = 0 to l S t = 4 sec

$$x = \left[\frac{1}{2}(10)(2)^2\right]_{(0\to 2)} + \left[(10\times 2)2 - \frac{1}{2}(10)(2)^2\right]_{(2\to 4)}$$

x = 40 m

$$y = \left[-\frac{1}{2}5(2)^2\right]_{(0\to 2)} - \left[(10(2) - \frac{1}{2}(10)(2)^2\right]_{(2\to 4)}$$

y = – 10 m

Hence, average velocity of particle between t = 0 to t = 4 sec is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\sqrt{(40)^2 + (-10)^2}}{4}$$

$$v_{av} = \frac{5}{2}\sqrt{17} \text{ m/s}$$

At t = 2 sec u = 10 × 2 = 20 m/s

After t = 2sec

v = u + at

0 = 20 – 10t

t = 2 sec.

Hence, at t = 4 sec. the particle is at its farthest distance from the y-axis.

The particle is at farthest distance from y-axis at t $\geq$ 4. Hence the available correct choice is t = 4.

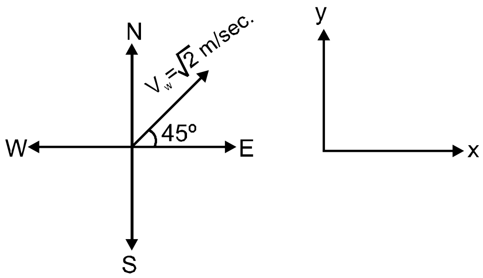
DPP NO. - 20

1. If speed of a particle changes, the velocity of the particle definitely changes and hence the acceleration of the particle is nonzero.

Velocity of a particle change without change in speed.

When speed of a particle varies, its velocity cannot be constant.

2. $V_w = 1\hat{i} + 1\hat{j}$



V = at

V = (0.2) 10

= 2 m/sec.

$V_{boat} = 2\hat{i} + 2\hat{j}$

$V_{w/boat} = V_w - V_{boat}$

$V_{w/boat} = (1\hat{i} + 1\hat{j}) - (2\hat{i} + 2\hat{j}) = -1\hat{i} - 1\hat{j}$

So, the flag will flutter towards south–west.

3. The retardation is given by

$$\frac{dv}{dt} = -av^2$$

integrating between proper limits

$$\Rightarrow -\int_u^v \frac{dv}{v^2} = \int_0^t a\,dt \quad \text{or} \quad \frac{1}{v} = at + \frac{1}{u}$$

$$\Rightarrow \frac{dt}{dx} = at + \frac{1}{u} \quad \Rightarrow \quad dx = \frac{u\,dt}{1+aut}$$

integrating between proper limits

$$\Rightarrow \int_0^s dx = \int_0^t \frac{u\,dt}{1+aut} \quad \Rightarrow \quad S = \frac{1}{a}\ln(1 + aut)$$

4. V = a + bx

(V increases as x increases)

$$\frac{dV}{dt} = b\frac{dx}{dt} = bV$$

hence acceleration increases as V increases with x.

6. $\vec{v} = -\hat{i} + \hat{j} + 2\hat{k}$

$\vec{a} = 3\hat{i} - \hat{j} + \hat{k}$

$\vec{a}.\vec{v} = -3 - 1 + 2 < 0$

hence $\theta > 90°$ between $\vec{a}$ and $\vec{v}$

so speed is decreasing

$\vec{a}.\vec{v} = -3 - 1 + 2 < 0$

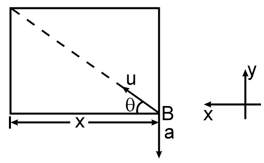
7. Solving the problem in the frame of train. Taking origin as corner 'B'

Along x axis x-

x = u cosθt ....(1)

Along y axis y-

$$y = u_y t + \frac{1}{2}a_y t^2$$



$$0 = u\sin\theta\, t - \frac{1}{2}at^2 \quad ....(2)$$

As the ball is thrown towards 'D'

$$\tan\theta = \frac{\ell}{x} \quad .....(3)$$

From equation (1), (2) & (3) we get

$$\mathbf{t} = \sqrt{\frac{2\ell}{a}}$$ required time after which ball hit the corner.

8. At position A balloon drops first particle So, $u_A = 0$, $a_A = -g$, t = 3.5 sec.

$$S_A = \left(\frac{1}{2}gt^2\right) \quad . \quad ....(i)$$

Balloon is going upward from A to B in 2 sec.so distance travelled by balloon in 2 second.

$$\left(S_B = \frac{1}{2}a_B t^2\right) \quad ....(ii)$$

$a_B = 0.4\ m/s^2$, $t = 2$ sec.

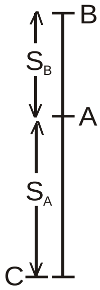
$S_1 = BC = (SB + SA)$ ....(iii)

Distance travell by second stone which is droped from balloon at B

$u_2 = u_B = a_B t = 0.4 \times 2 = 0.8$ m/s

$t = 1.5$ sec.

$\left(S_2 = u_2 t - \frac{1}{2} g t^2\right)$ ....(iv)

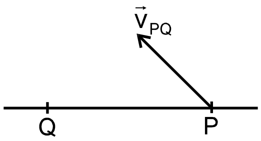


Distance between two stone

$\Delta S = S_1 - S_2$.

DPP NO. - 21

1.



Q measures acceleration of P to be zero.

$\therefore$ Q measures velocity of P, i.e. $\vec{v}_{PQ}$, to be constant. Hence Q observes P to move along straight line.

$\therefore$ For P and Q to collide Q should observe P to move along line PQ.

Hence PQ should not rotate.

2. Let initial and final speeds of stone be u and v.

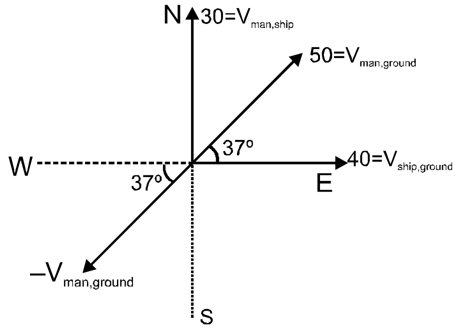
$\therefore$ $v^2 = u^2 - 2gh$ ....(1)

and $v \cos 30° = u \cos 60°$ ....(2)

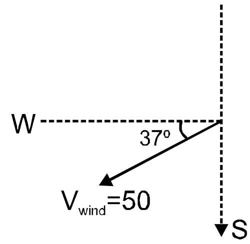
solving 1 and 2 we get $u = \sqrt{3gh}$

3. Flag will flutter in the direction of wind and opposite to the direction of velocity of man

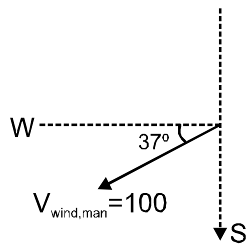
i.e. in the direction of V_{wm}



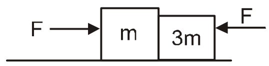
+



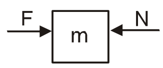
=



4. (i)

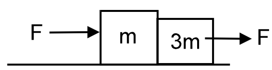


a = 0

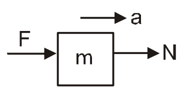


N = F.

(ii)

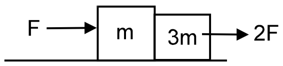


$a = \frac{2F}{4m} = \frac{F}{2m}$

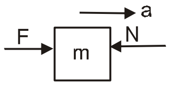


$F - N = ma$

$N = F - m\left(\frac{F}{2m}\right) = \frac{F}{2}$.

(iii) 

$a = \dfrac{3F}{4m}$

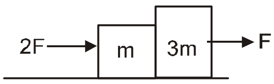


F – N = ma

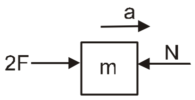
N = F – ma

$N = F - m\left(\dfrac{3F}{4m}\right)$

$N = \dfrac{F}{4}$.

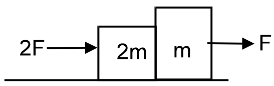
(iv) 

$a = \dfrac{3F}{4m}$

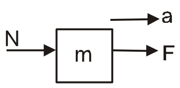


2F – N = ma $N = 2F - m\left(\dfrac{3F}{4m}\right)$

$N = \dfrac{5F}{4}$.

(v) 

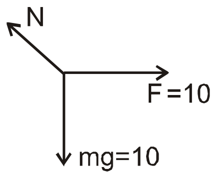
$a = \dfrac{3F}{3m} = \dfrac{F}{m}$



N + F = ma $N + F = m\left(\dfrac{F}{m}\right)$

N = 0.

5. F.B.D. of block



$N^2 = F^2 + (mg)^2$

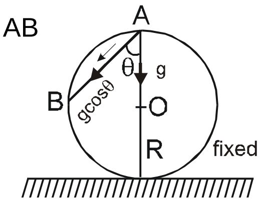
$N = 10\sqrt{2}$ N

6. AB = 2 R cosθ

acceleration along AB

a = g cos θ

u = 0 from A to B



$S = ut + \dfrac{1}{2}at^2$

$2R\cos\theta = 0 + \dfrac{1}{2}(g\cos\theta)\,t^2$

$t = 2\sqrt{\dfrac{R}{g}}$

7. Unit vector in direction of (1,0,0) to (4,4,12) is

$\dfrac{(4-1)\hat{i} + (4-0)\hat{j} + (12-0)\hat{k}}{13}$

Hence position of particle at t = 2 sec is :

$\vec{r}_f = \vec{r}_i + \vec{v} \times 2 = 31\hat{i} + 40\hat{j} + 120\hat{k}$

8. $a = \dfrac{F}{m}$ $V^2 = u^2 + 2as$ (u = 0)

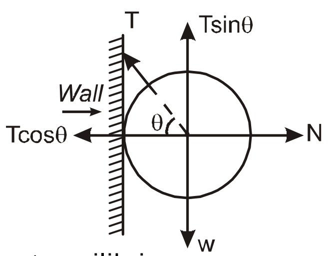
$V \propto \sqrt{2\left(\dfrac{F}{m}\right)S}$ $V \propto \dfrac{1}{\sqrt{m}}$.

DPP NO. - 22

1. From geometry :

$\cos\theta = \dfrac{3}{5}$

$\sin\theta = \dfrac{4}{5}$



As sphere is at equilibrium,

T sinθ = w

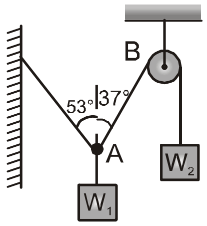
$T\left(\dfrac{4}{5}\right) = w$

$T = \dfrac{5w}{4}$.

2. Resolving forces at point A along string AB

$w_1 \cos 37° = w_2$

$$\frac{w_1}{w_2} = \frac{5}{4}$$



3. $v = 0 \Rightarrow x^2 - 5x + 4 = 0$

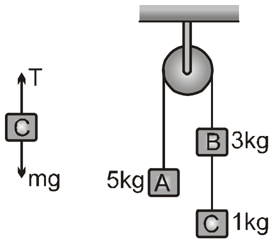
$x = 1m \ \& \ 4m$

$$\frac{dv}{dt} = (2x - 5)\, v = (2x - 5)\,(x^2 - 5x + 4)$$

at $x = 1$ m and 4m ; $\frac{dv}{dt} = 0$

4. $a = \left(\frac{5-4}{5+4}\right) g = \frac{g}{9}$

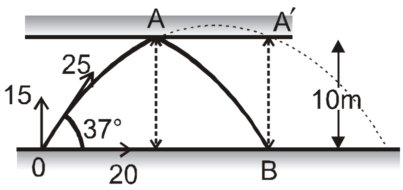
$T - mg = ma$



$T = m(g + a)$

$= 1\left(g + \frac{g}{9}\right) = \frac{10g}{9}$.

5. Time taken by ball from O to A is same as that from A to B.



$10 = 15\,t - \frac{1}{2}(10)\,t^2$

$5t^2 - 15\,t - 10 = 0$

$t^2 - 3t - 2 = 0$

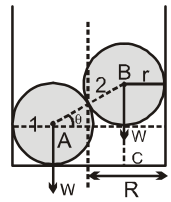
$t = 1, 2$

t = 2 is invalid as it is the time taken by the ball to come at A' if there was no roof.

$\therefore$ t = 1 seconds.

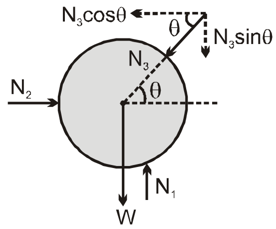
During this the ball will travel V × t = 20 × 2 = 40 m on the floor.

6.



r = 5cm ; R = 8cm

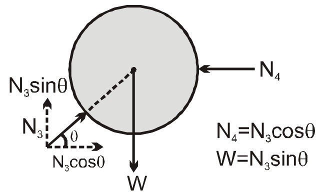
FBD of sphere 1



$N_1 = W + N_3 \sin\theta$

$N_2 = N_3 \cos\theta$

FBD of sphere 2



$AC = 2R - 2r$

$AB = 2r$

$$\cos\theta = \frac{AC}{AB} = \frac{R - r}{r}$$

$N_4 = N_3 \cos\theta$

$W = N_3 \sin\theta$

Ans. $N_4 = W \cot\theta$

$N_3 = W \operatorname{cosec}\theta$

$N_2 = W \cot\theta$

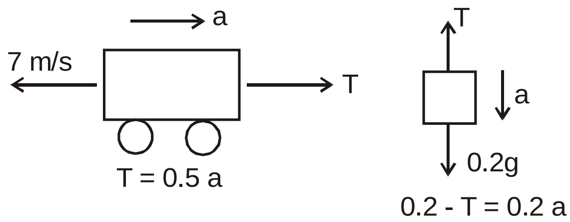
$N_1 = 2W$.

7. $\Rightarrow \ 0.2\, g = 0.7\, a$

$\Rightarrow a = \frac{2g}{7}$ m/s^2

For the case, it comes to rest when V = 0

$$0 = 7 + \left(-\frac{2g}{7}\right)t \quad \Rightarrow \quad t = \frac{49}{2g} = 2.5 \text{ s}$$



Distance travelled till it comes to rest

$$0 = 7^2 + 2\left(-\frac{2g}{7}\right)s$$

S = 8.75 m

So in next 2.5s, it covers 8.75 m towards right.

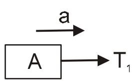
Total distance = 2 x 8.75 = 17.5 m

After 5s, it speed will be same as that of initial (7 m/s) but direction will be reversed.

8. Acceleration of system $a = \frac{F}{m_A + m_B + m_C}$

$$a = \frac{60}{10 + 20 + 30} = 1\text{m/s}^2$$

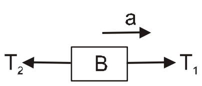
FBD of A :



$T_1 = m_A.a$

$T_1 = 10(1) = 10N$

FBD of B :



$T_2 - T_1 = m_B a$

$T_2 - 10 = 20(1)$

$T_2 = 30$ N.